Name: ______________________________

Instructor: __________________________

Math 10550, Practice Exam I Solutions
February 11, 3026

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 17 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-15.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice ______________

13. ______________

14. ______________

15. ______________

Total ______________

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Multiple Choice

1.(7 pts.) Compute

$$\lim_{x\to 2} \frac{\sqrt{x^2+21}-5}{x-2}.$$

Solution:

$$\begin{aligned}
\lim_{x\to 2} \frac{\sqrt{x^2+21}-5}{x-2} &= \lim_{x\to 2} \frac{\sqrt{x^2+21}-5}{x-2}\frac{\sqrt{x^2+21}+5}{\sqrt{x^2+21}+5} \\
&= \lim_{x\to 2} \frac{x^2-4}{(x-2)(\sqrt{x^2+21}+5)} \\
&= \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)(\sqrt{x^2+21}+5)} \\
&= \lim_{x\to 2} \frac{x+2}{\sqrt{x^2+21}+5} \\
&= \frac{4}{\sqrt{25}+5} \\
&= \frac{4}{10} \\
&= \frac{2}{5}.
\end{aligned}$$

(a) $\frac{2}{5}$ (b) 0 (c) $\frac{1}{10}$ (d) $\frac{1}{120}$ (e) 4

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2.(7 pts.) Alejandro runs a 40 meter dash. After t seconds of running, he is located $\frac{1}{3}t^3 + 4t$ meters away from the starting line. What is his average velocity over the time interval from $t = 2$ to $t = 3$ seconds? All answers are given in meters per second.

Solution

$$\begin{aligned}
\text{Ave. Velocity} &= \frac{(\text{pos. at } t=3) - (\text{pos. at } t=2)}{3-2} \\
&= \frac{\left(\frac{1}{3}(3)^3 + 12\right) - \left((\frac{1}{3}(2)^3 + 8\right)}{3-2} \\
&= (9+12) - \left(\frac{8}{3} + 8\right) \\
&= 13 - \frac{8}{3} \\
&= 10\frac{1}{3} \text{ or } \frac{31}{3}
\end{aligned}$$

(a) $31\frac{2}{3}$ $\left(\text{i.e., } \frac{95}{3}\right)$

(b) $2\frac{1}{15}$ $\left(\text{i.e., } \frac{31}{15}\right)$

(c) $10\frac{1}{3}$ $\left(\text{i.e., } \frac{31}{3}\right)$

(d) $1\frac{1}{2}$ $\left(\text{i.e., } \frac{3}{2}\right)$

(e) $26\frac{1}{3}$ $\left(\text{i.e., } \frac{79}{3}\right)$

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3.(7 pts.) Compute $\displaystyle\lim_{x\to -1^-} \frac{x^2+x}{x^2+2x+1}$

Solution: First, note that $\displaystyle\lim_{x\to -1^-} \frac{x^2+x}{x^2+2x+1} = \lim_{x\to -1^-} \frac{x(x+1)}{(x+1)(x+1)} = \lim_{x\to -1^-} \frac{x}{x+1}$. Now if you plug in $x=-1$ you get $\frac{-1}{0}$. Now, since we are looking at the limit from the left, we must argue if the limit goes to $+\infty$ or $-\infty$. Indeed the limit of the numerator is the constant -1. Additionally, the when approaching -1 from the left, the denominator takes on negative values closer and closer to 0. So $\displaystyle\lim_{x\to -1^-} \frac{x^2+x}{x^2+2x+1} = +\infty$.

(a) $-\infty$

(b) -1

(c) 0

(d) Does not exist and is not ∞ or $-\infty$.

(e) $+\infty$

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4.(7 pts.) Let $f(x) = \sqrt{2x^2+1}$. Which of the following limits equals $f'(2)$?

Solution: Note that for any a we have $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$. Now, $f(2) = \sqrt{2(2^2)+1} = \sqrt{9} = 3$. So $f'(2) = \lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2} \frac{\sqrt{2x^2+1}-3}{x-2}$.

(a) $\lim_{h\to 2} \frac{\sqrt{2(x+h)^2+1}-\sqrt{2x^2+1}}{h}$

(b) $\lim_{x\to 2} \frac{\sqrt{2x^2+1}-3}{x-2}$

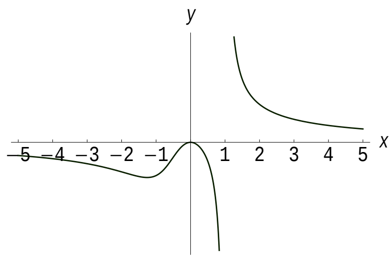
(c) $\lim_{x\to 0} \frac{\sqrt{2x^2+1}-3}{x}$

(d) $\lim_{h\to 0} \frac{\sqrt{2(x+h)^2+1}-\sqrt{2x^2+1}}{h}$

(e) $\lim_{h\to 2} \frac{\sqrt{2(x+h)^2+1}-3}{h}$

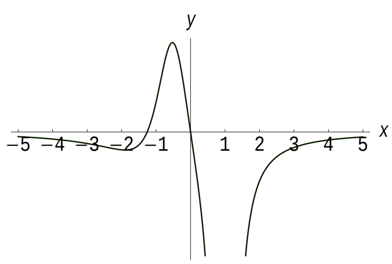
5.(7 pts.)

The graph of $f(x)$ is shown below:

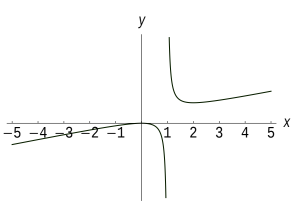


Which of the following is the graph of $f'(x)$?

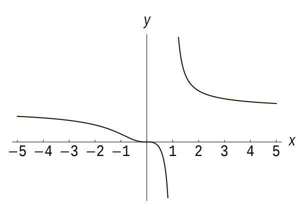
Solution: We observe that $f(x)$ has two horizontal tangent lines. One has the $x-$coordinate located somewhere in the interval [-2,-1] and the other one at $x = 0$. Hence $f'(x)$ has two zeroes at these two locations. Only this graph has that property.



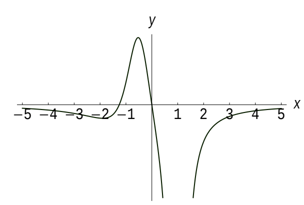
(a)



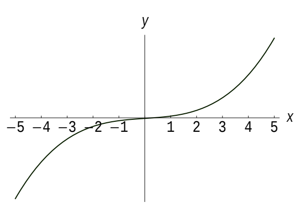
(b)



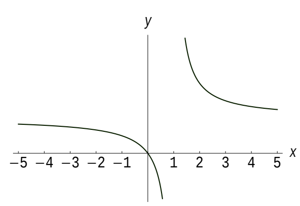
(c)



(d)



(e)



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6.(7 pts.) Evaluate: $\lim_{h\to 0} \frac{(1+h)^2 + 2(1+h) - 3}{h}$

Solution:

$$\begin{aligned}
\lim_{h\to 0} \frac{(1+h)^2 + 2(1+h) - 3}{h} &= \lim_{h\to 0} \frac{1 + 2h + h^2 + 2 + 2h - 3}{h} \\
&= \lim_{h\to 0} \frac{4h + h^2}{h} \\
&= \lim_{h\to 0} \frac{h(4+h)}{h} \\
&= \lim_{h\to 0}(4+h) \\
&= 4
\end{aligned}$$

(a) 1

(b) 2

(c) 0

(d) 4

(e) The limit does not exist (DNE)

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7.(7 pts.) For what values of c is the function f given by

$$f(x) = \begin{cases} x^2 + c^2 x - 3 & x < 2 \\ \\ cx + 5 & x \geq 2 \end{cases}$$

continuous at $x = 2$?

Solution: In order for $f(x)$ to be continuous at $x = 2$ we need

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2).$$

We have $\lim_{x \to 2^-} f(x) = 2^2 + 2c^2 - 3$ and $f(2) = 2c + 5$, and so $2^2 + 2c^2 - 3 = 2c + 5$. Therefore $c = 2, -1$.

(a) No value of c makes f continuous at $x = 2$

(b) $c = 2$ only

(c) $c = 1$ only

(d) $c = 0$ only

(e) $c = 2$ and $c = -1$

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8.(7 pts.) For what value(s) of a and b is $f(x)$ continuous and differentiable at $x = 1$?

$$f(x) = \begin{cases} x^2 + ax + 2 & \text{if } x < 1 \\ bx^2 - 1 & \text{if } x \geq 1 \end{cases}$$

Solution: For continuity at $x = 1$ we need

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

i.e. we need $(1)^2 + a(1) + 2 = b(1)^2 - 1$ which gives $a + 3 = b - 1$ or $\boxed{b = a + 4}$.

For differentiability at $x = 1$ we need the derivatives of both pieces to match at $x = 1$, i.e. we need $2(1) + a = 2b(1)$ or $\boxed{a = 2b - 2}$.

Putting the two pieces of information together, we get $b = a + 4 = (2b - 2) + 4 = 2b + 2$. This gives $\boxed{b = -2}$.

Substituting this into the equation $a = 2b - 2$, we get $\boxed{a = -6}$.

(a) $a = -1,\ b = -4$ (b) $a = -6,\ b = -2$ (c) $a = -4,\ b = -1$

(d) $a = -1,\ b = 3$ (e) $a = 0,\ b = 4$

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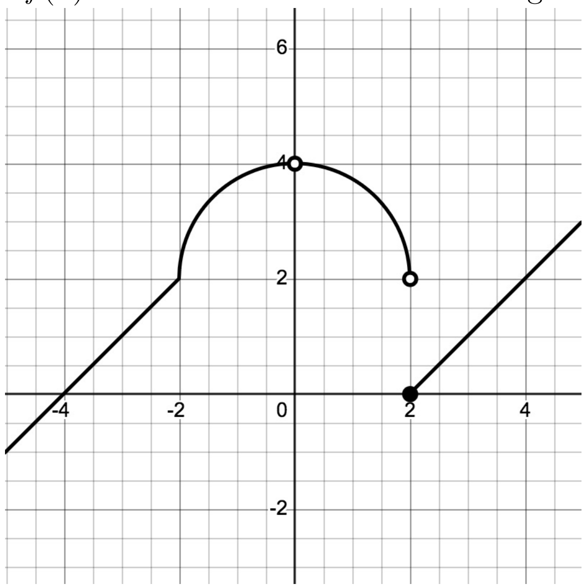
9.(7 pts.) The graph of $f(x)$ is shown. Which of the following statements is true?



Solution $f(x)$ is continuous from the right at $x = 2$, since $f(2) = 0 = \lim_{x \to 2^+} f(x)$.

$f(x)$ has a jump discontinuity at $x = 2$, which is not removable.

$f'(0)$ does not exist since $f(x)$ is not continuous at $x = 0$

$f'(-2)$ does not exist since the graph of $f(x)$ has a sharp corner at $x = -2$

$\lim_{x \to 0} f(x) = 4$.

(a) $f'(-2) = 1$

(b) $f'(0) = 0$

(c) $f(x)$ is continuous from the right at $x = 2$.

(d) $\lim_{x \to 0} f(x)$ does not exist

(e) $f(x)$ has a removable discontinuity at $x = 2$.

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10.(7 pts.) Evaluate $\lim_{x \to -\infty} \dfrac{3x^3 - 2x + 1}{2x^2 + x + 1}$

Solution

$$\lim_{x \to -\infty} \frac{3x^3 - 2x + 1}{2x^2 + x + 1} = \lim_{x \to -\infty} \frac{\frac{3x^3}{x^3} - \frac{2x}{x^3} + \frac{1}{x^3}}{\frac{2x^2}{x^3} + \frac{x}{x^3} + \frac{1}{x^3}} = \lim_{x \to -\infty} \frac{3 - \frac{2}{x^2} + \frac{1}{x^3}}{\frac{2}{x} + \frac{1}{x^2} + \frac{1}{x^3}} = -\infty.$$

(a) $\frac{3}{2}$ (b) 0 (c) $-\frac{3}{2}$

(d) $-\infty$ (e) Does not exist

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11.(7 pts.) Evaluate $\lim_{x \to -\infty} \dfrac{\sqrt{4x^6+5}}{x^3+1}$.

Solution: We first divide through by x^3, in the numerator using the fact that $\sqrt{x^6} = -x^3$ for $x < 0$:

$$\frac{\sqrt{4x^6+5}}{x^3+1} = -\frac{\sqrt{4+5/(x^6)}}{1+1/(x^3)}.$$

As x goes to $-\infty$, the numerator goes to $\sqrt{4} = 2$. As x goes to $-\infty$, the denominator goes to 1. Hence the answer is -2.

(a) $3/2$ (b) 6 (c) -2 (d) 4 (e) 2

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12.(7 pts.) If $f(x) = x^3 - 3x^2 - 9x + 7$, find the x-coordinates of all points on the curve with horizontal tangent line.

Solution: We solve $f'(x) = 3x^2 - 6x - 9 = 0$. So $x = 3, -1$.

(a) $x = 0$ and $x = 1$

(b) $x = 3$ and $x = -1$

(c) $x = 4$ and $x = -2$

(d) $x = -3$ and $x = 1$

(e) No points on the curve have horizontal tangent line.

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Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(13 pts.) Find the derivative of

$$f(x) = \sqrt{x+1}$$

using the limit definition of the derivative.
Please include all of the details in your calculation.

Solution:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} * \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}} \\
&= \lim_{h\to 0} \frac{(x+h+1) - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\
&= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} \\
&= \frac{1}{2\sqrt{x+1}}
\end{aligned}$$

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14.(13 pts.) Show that there is at least one solution of the equation

$$x^2 = 2 + \sin(\pi x).$$

Justify your answer, identify the theorem you use and explain why the theorem applies.

Solution:

First, note that finding a solution to $x^2 = 2 + \sin(\pi x)$ is equivalent to finding the zeros to the function $f(x) = 2 + \sin(\pi x) - x^2$. Indeed, $f(x)$ is continuous since 2 ,$\sin(\pi x)$ and $-x^2$ are all continuous and the sum of continuous functions is again continuous.

Further note that $f(0) = 2 + \sin(0) - 0^2 = 2$ and $f(2) = 2 + \sin(2\pi) - 4 = -2$.

The intermediate value theorem states that for any continuous function on an interval $[a, b]$ and a number N between $f(a)$ and $f(b)$ where $f(a) \neq f(b)$ there is a number $c \in (a, b)$ such that $f(c) = N$. We can apply the IVT to our case and conclude that since $f(2) = -2 < 0 < 2 = f(0)$ there is some $c \in (0, 2)$ such that $f(c) = 0$.

Indeed for that c, we will have $c^2 = 2 + \sin(c\pi)$.

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15.(4 pts.) Please circle "TRUE" if you think the statement is true, and circle "FALSE" if you think the statement is False.

(a)(1 pt. No Partial credit).) $\lim_{x\to 0^+} \ln(x) = 1$.

FALSE $\lim_{x\to 0^+} \ln(x) = -\infty$

TRUE FALSE

(b)(1 pt. No Partial credit) If the function $f(x)$ is continuous at $x = a$, then it must be differentiable at $x = a$

FALSE Consider $f(x) = |x|$ at $x = 0$.

TRUE FALSE

(c))(1 pt. No Partial credit).)

If $f(x) = 4x^3 + 2x^2 + x + 1$, then $f''(1) = 30$.

FALSE $f'(x) = 12x^2 + 4x + 1,\ \ f''(x) = 24x+,\ f''(1) = 28.$

TRUE FALSE

(d))(1 pt. No Partial credit) $\frac{\pi}{2}$ is in the domain of the function $f(x) = \tan(x)$.

FALSE $\tan(x) = \sin(x)/\cos(x)$, at $x = \frac{\pi}{2}$, $\cos(x) = 0$ and $\sin(x) = 1$, therefore $\tan(x)$ is not defined at $x = \frac{\pi}{2}$.

TRUE FALSE

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Rough Work

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3.	(a)	(b)	(c)	(d)	(•)
4.	(a)	(•)	(c)	(d)	(e)
5.	(a)	(b)	(•)	(d)	(e)
6.	(a)	(b)	(c)	(•)	(e)
7.	(a)	(b)	(c)	(d)	(•)
8.	(a)	(•)	(c)	(d)	(e)
9.	(a)	(b)	(•)	(d)	(e)
10.	(a)	(b)	(c)	(•)	(e)
11.	(a)	(b)	(•)	(d)	(e)
12.	(a)	(•)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice ______________

13. ______________

14. ______________

15. ______________

Total ______________